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# The choice of type of input-output table revisited: moving towards the use of supply-use tables in impact analysis

José M. Rueda-Cantuche[1,*]

*European Commission, Joint Research Centre-IPTS and Pablo de Olavide University at Seville*

**Abstract**

The construction of symmetric input-output tables (SIOTs) is a controversial issue as regards the choice of model to construct both product-by-product and industry-by-industry SIOTs, especially the former ones. However, there has been little attention paid so far by the UN and the Eurostat Systems of National Accounts on the choice of type of SIOT to carry out impact analyses let alone other input-output applications. Concerning the price and quantity models in input-output analysis, this paper identifies severe problems in the correct interpretation of the meaning of their results and proposes the use of supply and use tables instead of SIOTs to solve these problems.

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*Keywords:* Statistical methods, economic indices and measures, multisectoral models.

## 1. Introduction

Typical research questions that can be addressed by input-output analysis are as follows. What is the impact on employment of an increase in households' consumption of renewable energies? Or what would be the effect on fuel prices of an increase in the labour costs of the electricity industry? Many input-output practitioners would claim that they could easily answer these questions as long as they could dispose of the so-called symmetric input-output tables (SIOTs). However, very few authors reflect on the issue

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[1] European Commission, Joint Research Centre, Institute for Prospective Technological Studies (IPTS) and Pablo de Olavide University at Seville. Phone: +34 954488363. E-mail: Jose.Rueda-Cantuche@ec.europa.eu

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that in both examples the impact drivers and the resulting effects are referring to different issues. On the one hand, households may increase their consumption of bio-fuels (of a single product or group of products) while the impacts actually refer to the number of jobs created in a certain industry. On the other hand, labour costs have increased in the electricity industry while the price effects should refer to a single product (e.g. fossil fuel). Thus, we believe that the main unnoticed shortcoming underlying the use of SIOTs to address these types of research questions is precisely its symmetry, in the sense that they are defined either on a product-by-product or on an industry-by-industry basis. Moreover, although the choice of type of SIOT is playing increasingly a relevant role in the most recent systems of national accounts, they still provide unclear guidelines on the type of table to be used for what type of analysis. There is no clear structure or even clear recommendations. As it will be shown in this paper, the so-called supply and use tables solve efficiently this matter since they are defined on a product-by-industry basis rather than on a product or on an industry basis only. Therefore, we will eventually recommend exploring new possibilities in order to find suitable supply-use based input-output techniques to give a proper answer to the type of questions raised at the beginning of this paragraph.

Accordingly, Section 2 will introduce the input-output framework; the next section will address how the issue of the choice of type of SIOT is insufficiently dealt with by the United Nations and European Systems of National Accounts. Section 4 reviews the most relevant input-output applications, namely the quantity and the price models; and the last Section concludes with some recommendations on the benefits of using supply and use tables rather than SIOTs in impact input-output analysis.

## 2. The input-output framework

Following Rueda-Cantuche *et al.* (2009), an input-output framework revolves around the so-called supply and use tables. They can be seen as the output mix of industries and the industries' use of inputs, respectively. On the one hand, the supply table comprises an intermediate matrix of goods and services (rows) produced by industries (columns), plus additional column vectors including imports, distribution margins (trade and transport) and net taxes on products, all of which make the total supply of products of an economy. On the other hand, the use table represents domestically produced and imported intermediate and final uses. They may be valued at basic and at purchasers' prices. There are several additional column vectors that show the usual final demand categories, i.e. final consumption, investment and exports; and additional rows, which eventually represent the different components of the gross value added, e.g. labour costs, capital use, other net taxes on production and net operating surplus (see Tables 1 and 2).

Note that the valuation of the supply and use tables is not coincident. The supply table is measured at basic prices, which means excluding trade and transport margins and net taxes on products. To the contrary, the use table is measured at purchasers' prices,

***Table 1:*** *Simplified overview of a supply table (Rueda-Cantuche* et al.*, 2009).*

| | INDUSTRIES (NACE) / PRODUCTS (CPA) | OUTPUT OF INDUSTRIES (NACE) | | | | | | | IMPORTS | | | Total supply at basic prices | VALUATION | | Total supply at purcha-sers' prices |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Agriculture | Industry | Construction | Trade | Private services | Government services | Total | Intra EU imports cif | Extra EU imports cif | Imports cif | | Trade and transport margins | Taxes less subsidies on products | |
| No | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | Products of agriculture | Production matrix ($V^T$) | | | | | | | Imports cif | | | | Valuation items | | |
| 2 | Products of industry | | | | | | | | | | | | | | |
| 3 | Construction work | | | | | | | | | | | | | | |
| 4 | Trade | | | | | | | | | | | | | | |
| 5 | Private services | | | | | | | | | | | | | | |
| 6 | Government services | | | | | | | | | | | | | | |
| 7 | Total | | | | | | | | | | | | | | |
| 8 | Cif/ fob adjustments on imports | | | | | | | | | | | | | | |
| 9 | Direct purchases abroad by residents | | | | | | | | | | | | | | |
| 10 | Output at basic prices | Total output of industries at basc prices | | | | | | | Total imports | | | | Total | | |

***Table 2:*** *Simplified overview of a use table (Rueda-Cantuche* et al.*, 2009).*

| | INDUSTRIES (NACE) / PRODUCTS (CPA) | OUTPUT OF INDUSTRIES (NACE) | | | | | | | FINAL USES | | | | | | | | | Total use at purchasers' prices |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Agriculture | Industry | Construction | Trade | Private services | Government services | Total | Final consumption expenditure by households | Final consumption expenditure by non-profit organisations | Final consumption expenditure by government | Gross fixed capital formation | Changes in valuables | Changes in inventories | Exports intra EU fob | Exports extra EU fob | Total | |
| No | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 1 | Products of agriculture | Intermediate uses (U) | | | | | | | Final uses (Y) | | | | | | | | | |
| 2 | Products of industry | | | | | | | | | | | | | | | | | |
| 3 | Construction work | | | | | | | | | | | | | | | | | |
| 4 | Trade | | | | | | | | | | | | | | | | | |
| 5 | Private services | | | | | | | | | | | | | | | | | |
| 6 | Government services | | | | | | | | | | | | | | | | | |
| 7 | Total | Total intermediate consumption | | | | | | | Total final uses of goods and services | | | | | | | | | |
| 8 | Cif/ fob adjustments on exports | | | | | | | | | | | | | | | | | |
| 9 | Direct purchases abroad by residents | | | | | | | | | | | | | | | | | |
| 10 | Domestic purchases. by non-residents | | | | | | | | | | | | | | | | | |
| 11 | Total | Total intermediates | | | | | | | Total final uses | | | | | | | | | |
| 12 | Compensation of employees | Value added (W) | | | | | | | | | | | | | | | | |
| 13 | Other net taxes on production | | | | | | | | | | | | | | | | | |
| 14 | Consumption of fixed capital | | | | | | | | | | | | | | | | | |
| 15 | Operating surplus, net | | | | | | | | | | | | | | | | | |
| 16 | Total | Total value added at basic prices | | | | | | | | | | | | | | | | |
| 17 | Output at basic prices | Total output of industries at basic prices | | | | | | | | | | | | | | | | |

which means at the price producers and/or consumers pay goods and services for final use or intermediate inputs (including trade and transport margins and taxes less subsidies on products). As stated by Eurostat (2008), basic prices are the preferable valuation concept in the supply and use framework in the sense that it provides a more homogeneous valuation. Thus, for analytical purposes a valuation as much homogeneous as possible will be required as the input-output relations are to be interpreted as technical coefficients.

The construction of SIOTs has suffered from controversial contributions in the literature. On the one hand, a product-by-product table describes the technological relations between products (Eurostat, 2008). The intermediate matrix describes a kind of recipe of how to produce commodities in terms of the amounts used of others, irrespective of the producing industry. On the other hand, industry-by-industry tables depict inter-industry relations. The intermediate matrix would describe on an industry basis, the use of commodities of the other industries (Eurostat, 2008).

Independently of the purpose of the analysis, both types of SIOTs have their own advantages and disadvantages. On the one hand, the product-by-product tables are more homogeneous in their description of the transactions being one of the most commonly used tables in input-output analysis (productivity, comparison of costs structures, employment effects, energy policy...) and have a clear input structure in terms of products for intermediate uses and value added for the compensation of labour and capital for homogenous branches. However, product-by-product tables require labour intensive compilation tasks; they must be based on analytical assumptions that take final results away from actual market transactions and observations, and hence, they make more difficult the integration of other statistical sources and the reporting on the transformation procedure. On the other hand, industry-by-industry tables are much closer to statistical sources; they allow for an easier comparability with other statistical databases; they are less labour intensive to compile, being based on pragmatic assumptions rather than on analytical hypotheses. Nevertheless, the larger the secondary activities in the supply table are the more difficult it becomes to identify homogeneous cost structures in an industry-by-industry table.

In practice, most of the countries worldwide compile product-by-product tables although there are some hardly negligible countries like Denmark, the Netherlands, Norway, Canada and Finland that compile industry-by-industry SIOTs. Nevertheless, one can always shift from one type to another as it is shown in Table 3.

Basically, the choice of the type of SIOT is related to the treatment of secondary products (Rueda-Cantuche and ten Raa, 2009). There are two main approaches to eliminating secondary production from industries in order to get homogenous branches of production in a product-by-product SIOT. Both of them can be derived from combining the information on input structures depicted by the use table at basic prices with the supply table so that all the secondary production (including the inputs used to produce them) are re-allocated either to the industry for which the product is a primary output (product technology, Model A) or to the main product of the industry that actually produces it (industry technology, Model B). The transformed use table is what is referred to as an input-output table (UN, 2009, par. 28.47). It follows that in deriving a product-by-product matrix in the simplest possible way, the final demand of the use table remains unchanged. By contrast, the demand for intermediate uses and labour and capital inputs are determined by the nature of the products made (UN, 2009, par. 28.48).

There are other possible technology assumptions available in the literature, that were reviewed by Viet (1994) and by ten Raa and Rueda-Cantuche (2003), who also provided their advantages and disadvantages from a theoretical approach (see also Kop Jansen and ten Raa, 1990). For more details, the interested reader could check the above references.

In deriving an industry-by-industry SIOT in the simplest way, the key issue is reallocating items between rows rather than between columns (as in product-by-product SIOTs). Contrarily to the product-by-product SIOTs, final uses will have to change thus indicating now the intermediate and final demand associated to the industry supplying the products rather than to the products themselves. Recall that the use tables have industries in columns and products in rows and we aim to construct a SIOT with industries both in rows and columns. Concerning the value added components, they remain unchanged because the level of the industry outputs will not be altered by the methods used for the construction of the SIOT.

It is assumed that as the level of the product output changes into that of the industry output, the pattern of sales will however remain the same. This is called a sales structure approach and only two approaches may be identified: the fixed industry sales structure assumption (Model C), where the industry deliveries are independent of the products delivered, and the fixed product sales structure (Model D), where they are instead independent of the producing industry. Rueda-Cantuche and ten Raa (2009) identified Model C as the most suitable from an axiomatic point of view.

For reading Table 3, let us define a use matrix, $\mathbf{U} = (u_{ij})\ i, j = 1, \ldots, n$ of products $i$ consumed by industry $j$, and a supply matrix $\mathbf{V}^{\mathsf{T}} = (v_{ij})\ i, j = 1, \ldots, n$ where product $i$ is produced by industry $j$, which is actually the transposition of the so-called make matrix $\mathbf{V}$. Models A, B, C and D can be easily formalized on the basis of supply and use matrices as it is shown in Table 3, where we provide bridges matrices that can be used to shift from one model to another. The matrices in the main diagonal refer to the mathematical expressions of the technical coefficient matrices of each model. Eventually, SIOTs can be calculated by post-multiplying the $\mathbf{A}$ matrices depicted in Table 3 with a diagonalised matrix of product outputs (for Models A and B) or of industry outputs (for Models C and D). Simple matrix algebra can be used by the reader to trace proofs.

Following the Eurostat Manual's (2008, p.349) notation and denoting as ˆ the diagonalization whether by suppression of the off-diagonal elements of a square matrix or by placement of the elements of a vector; we have denoted $\mathbf{g}$ as the column vector of industry output; $\mathbf{q}$ as the column vector of product output; $\mathbf{C} = \mathbf{V}^{\mathsf{T}}\hat{\mathbf{g}}^{-1}$ as the product-mix matrix with share of each product in industry outputs (supply table); $\mathbf{D} = \mathbf{V}\hat{\mathbf{q}}^{-1}$ as the market shares matrix with contribution of each industry to the product output (supply table); and $\mathbf{Z} = \mathbf{U}\hat{\mathbf{g}}^{-1}$ as the inputs requirements for products per unit of output of an industry (use table).

Product-by-product SIOTs (mainly using Model A or a slightly modified version) are the most common type of SIOT compiled by many European Union countries. Furthermore, Model B implies a mix of input structures that makes the use of product-by-product SIOTs inconsistent with technically oriented input-output analysis. Some European Union countries compile industry-by-industry SIOTs. They usually apply Model D (fixed product sales structure) for the transformation of supply and use tables into input-output tables. Model D is clearly preferred, due to the unrealistic feature of the alternative assumption of fixed industry sales structure.

## 3. The choice of type of input-output table in the UN and European systems of national accounts

The choice of technology assumption in the construction of product-by-product SIOTs has played a relevant role in the various systems of national accounts and handbooks/manuals published by the United Nations (UN) and Eurostat. To the contrary, the choice of type of SIOTs (product-by-product or industry-by-industry) has been almost fully neglected. In this section, we will explore the treatment of this issue by the two latest systems of national accounts published by the UN and Eurostat together with their respective handbooks or manuals.

### *3.1. SNA93, UN Handbook of IO Compilation (1999) and SNA08*

Essentially, the SNA93 (UN, 1993) states that only product-by-product tables will be described in detailed since they are often proved as most useful (par. 15.150) but however the SNA93 does not provide any justification for this assortment and simply ignores industry-by-industry tables.

It was not until the publication of the UN Handbook of Input-Output Compilation and Analysis (UN, 1999) when industry-by-industry tables received a more detailed treatment, although still not too far reaching. After providing the definitions of product and industry SIOTs (par. 4.41), the UN Handbook asserts that industry-by-industry SIOTs are much less useful than product-by-product SIOTs because an industry might represent a group of establishments, part of which may be artificially created by mathematical methods (e.g. extrapolation) and therefore, does not reflect any "realistic" picture of the economy. Concerning IO modeling, the UN Handbook (par. 4.60) also states that industry-by-industry tables are of almost no interest to analysts since final demand is, rarely, in terms of industry outputs.

With an increasing interest for industry-by-industry SIOTs, the new System of National Accounts-SNA08 (UN, 2009) now includes one section specifically for these

***Table 3:*** *Bridge matrices for technical coefficients to switch between different types of SIOTs.*

| To:<br><br>From: | MODEL A<br>Product-by-product<br>Product technology based | MODEL B<br>Product-by-product<br>Industry technology based | MODEL C<br>Industry-by-industry<br>Fixed industry sales structure | MODEL D<br>Industry-by-industry<br>Fixed product sales structure |
|---|---|---|---|---|
| Model A | $\mathbf{A_A (U, V) = Z\, C^{-1}}$ | $\mathbf{A_B (U, V) = A_A\, CD}$ | $\mathbf{A_C (U, V) = C^{-1} A_A\, C}$ | $\mathbf{A_D (U, V) = DA_A\, C}$ |
| Model B | $\mathbf{A_A (U, V) = A_B\, D^{-1} C^{-1}}$ | $\mathbf{A_B (U, V) = Z\, D}$ | $\mathbf{A_C (U, V) = C^{-1} A_B\, D^{-1}}$ | $\mathbf{A_D (U, V) = DA_B\, D^{-1}}$ |
| Model C | $\mathbf{A_A (U, V) = CA_C\, C^{-1}}$ | $\mathbf{A_B (U, V) = CA_C\, D}$ | $\mathbf{A_C (U, V) = C^{-1} Z}$ | $\mathbf{A_D (U, V) = DCA_C}$ |
| Model D | $\mathbf{A_A (U, V) = D^{-1} A_D\, C^{-1}}$ | $\mathbf{A_B (U, V) = D^{-1} A_D\, D}$ | $\mathbf{A_C (U, V) = C^{-1} D^{-1} A_D}$ | $\mathbf{A_D (U, V) = D\, Z}$ |

**Legend**
**A** = Technical coefficients matrix
$\mathbf{V}^T$ = Supply matrix
**U** = Use matrix
**e** = Column vector of ones
**Z** = Inputs requirements for products per unit of output of an industry (use table)
**C** = Product-mix matrix with share of each product in output of an industry (supply table)
**D** = Market shares matrix with contribution of each industry to the output of a product (supply table)

kinds of tables (pars. 28.57 to 28.63). As to the choice of type of SIOTs, the SNA08 states that both product-by-product and industry-by-industry SIOTs serve different analytical functions (price consistency, labour market, technology, inter-industry relations...). It is also interesting to remark that in one of the annexes (par. A4.21), the SNA08 recognizes a change of emphasis from product-by-product SIOTs to industry-by-industry ones.

### 3.2. ESA95, the Eurostat Manual of Supply, Use and IO Tables (2008) and the ESA08 (draft version)

Unfortunately, to the knowledge of the authors, neither the ESA95 nor the draft version of the European System of Accounts – ESA08 (Eurostat, 2009) – mentions explicitly the issue of the choice of type of SIOTs. The ESA95 just offers a flexible approach to compile industry-by-industry SIOTs or product-by-product SIOTs according to the objective of economic analysis. As in the SNA93, it is recommended to compile the latter tables although industry-by-industry tables are also accepted if the industries are close to homogenous units of production (Eurostat, 2008; p.31). Nevertheless, the Eurostat Manual (2008) considerably deals with this issue in its chapter 11.

Following the Eurostat Manual (2008, p. 301), (...) "*product-by-product input-output tables are theoretically more homogeneous in their description of the transactions than industry-by-industry tables, since a single element of the latter can refer to products that are characteristic in other industries. This supports the assumption that in practice product-by-product tables generally are better suited for many types of input-output analysis. For example, it seems more feasible to use product-by-product input-output tables for productivity analysis or the analysis of new technologies in the economy. On the other hand, industry-by-industry input-output tables are possibly the better option if the economic impact of a major tax reform is studied on the basis of input-output data* (...)". Similarly to the UN Systems of National Accounts (SNA93 and SNA08) , there is also here only a general remark on the suitability of the type of SIOT, which cannot be considered as a clear guidance on which types of tables are to be used for what type of analysis.

Broadly speaking, very little secondary output reported in the supply table would lead to fade away the distinction between products and industries. So, a relatively low level of secondary activities reported in the European Union supply tables may well suggest, as one can read in the Eurostat Manual (2008, p. 309), that the difference between product-by-product SIOTs and industry-by-industry SIOTs is relatively small, and consequently both transformations can be regarded as valid options for impact analysis. However, (...) "*it must be noticed that secondary activities vary considerably across sectors even the general level is low* (...)" (Eurostat, 2008; p. 309).

The Eurostat Manual (2008, p. 340) eventually argues that "*the type of tables that best fulfils the standard quality criteria is the industry-by-industry table based on the assumption of fixed product sales structures and the product-by-product SIOT based* *on the product technology assumption. These types of tables reflect the accumulated experience and current practice of those countries most permanently involved in the compilation of SIOTs*". Focusing on the these two models (Models A and D) to construct product-by-product tables and industry-by-industry tables, respectively, the Eurostat Manual defines a set of quality features of both types of SIOTs (p. 340-341):

*Transparency*

Industry-by-industry SIOTs provide more transparency than product-by-product SIOTs because the fixed product sales structure assumption can be derived from the supply and use tables without too much effort and in such a way that negatives do not appear. Conversely, the product technology assumption is usually applied in a complex context requiring a balancing procedure to treat the negative elements that may arise and thus, causing less transparency.

*Comparability*

Industry-by-industry SIOTs guarantee more comparability with national accounts data since they are closer to statistical sources, survey results and actual observations. To the contrary, product-by-product tables have been compiled in an analytical step which creates less comparability with the sources but at the same time guarantees more comparability across nations.

*Inputs*

Product-by-product SIOTs have a clear input structure in terms of products for intermediate use and value added for the compensation of labour and capital for homogenous branches. However, in industry-by-industry SIOTs, mixed bundles of goods and services rather than homogeneous products are reported for intermediate and final uses.

*Resources and timeliness*

The compilation of product-by-product tables based on the product technology assumption requires more resources and balancing efforts due to the treatment of the negatives that may appear. Consequently, publication may be delayed. However, industry-by-industry tables can be directly derived from supply and use tables with less resource intensive efforts.

*Analytical potential*

The Eurostat Manual (2008, p. 341) states that "*industry-by-industry tables are well suited for specific analytical purposes which are related to industries (tax reform, impact analysis, fiscal policy, monetary policy, etc.)*" while product-by-product tables "*are well suited for many other specific analytical purposes which are related to* *homogeneous production units (productivity, comparison of cost structures, employment effects, energy policy, environmental policy, etc.)*" Although useful, this distinction just enumerates possible applications without a clear guidance on which types of tables are to be used for what type of analysis, which will hopefully be provided by this deliverable.

To cut a long story short, the choice of type of SIOT is not a relevant issue in the two most recent ESAs (1995 and 2008) although the Eurostat Manual (2008) gives much more insight into the matter than any of the UN documents. However, we still think that a deeper and clearer connection between standard input-output applications and the use of product-by-product and/or industry-by-industry tables is needed.

## 4. The relevance of the applications: the quantity and the price models in input-output analysis

### *4.1. The quantity and price models in input-output analysis*

The main purpose of this section is to present briefly the theoretical background of the two most commonly and broadly used models in input-output analysis, i.e. the quantity and the price models. It will follow a discussion on the choice of type of SIOT for each type of model together with some guidelines.

Dietzenbacher (1997) considered the following SIOT in money terms (say, euros) for period 0:

| | | |
|---|---|---|
| $\mathbf{X}_0$ | $\mathbf{f}_0$ | $\mathbf{x}_0$ |
| $\mathbf{v}_0^{\mathsf{T}}$ | — | $\mathbf{v}_0^{\mathsf{T}}\mathbf{e}$ |
| $\mathbf{x}_0^{\mathsf{T}}$ | $\mathbf{e}^{\mathsf{T}}\mathbf{f}_0$ | |

$\mathbf{X}_0$ is the $n \times n$ matrix of intermediate uses; its typical element $x_{ij}^0$ denotes the value (in euros) of the deliveries from industry (product) $i$ to industry (product) $j$, which will depend on the type of SIOT used. Dietzenbacher (1997) did not however distinguish in his paper between the two types of SIOTs referring implicitly all the time to industry-by-industry tables. The column vector $\mathbf{f}_0$ can be interpreted as sectoral (product) final demands including private and government consumption, investments and net exports[1]. The row vector $\mathbf{v}_0^{\mathsf{T}}$ gives the value added in each industry (product or homogenous branch), containing, for instance, payments for the labour and capital primary factors. The value of each industry (product) output is given by the elements of the vector $\mathbf{x}_0$

1. Dietzenbacher (1997) made this assumption without loss of generality and for the sake of notational convenience.

while $\mathbf{e}$ denotes the *n*-dimension column vector of ones. Column-wise, a SIOT depicts input structures and row-wise, output structures. Since the total value of outputs equals the total value of inputs, for each industry (product), the following sets of accounting equations are obtained:

$$\mathbf{x}_0 = \mathbf{X}_0\,\mathbf{e} + \mathbf{f}_0 \tag{1}$$

$$\mathbf{x}_0^{\mathsf{T}} = \mathbf{e}^{\mathsf{T}}\,\mathbf{X}_0 + \mathbf{v}_0^{\mathsf{T}} \tag{2}$$

It follows that the input coefficients are defined as the industry (product) *i*'s input into industry (product) *j* as a fraction of the purchaser's output ($x_j^0$). They are obtained as $a_{ij}^0 = x_{ij}^0/x_j^0$, or in matrix terms, as $\mathbf{A}_0 = \mathbf{X}_0\,\hat{\mathbf{x}}_0^{-1}$ where $\hat{\mathbf{x}}_0$ denotes a diagonal matrix. Then, equation (1) may be written as:

$$\mathbf{x}_0 = \mathbf{A}_0\,\mathbf{x}_0 + \mathbf{f}_0 \tag{3}$$

In a similar way, the output coefficients denote the industry (product) *i*'s delivery to industry (product) *j* as a fraction of the seller's output ($x_i^0$). They are obtained as $b_{ij}^0 = x_{ij}^0/x_i^0$ or, in matrix terms, as $\mathbf{B}_0 = \hat{\mathbf{x}}_0^{-1}\,\mathbf{X}_0$. Subsequently, equation (2) may be rewritten as

$$\mathbf{x}_0^{\mathsf{T}} = \mathbf{x}_0^{\mathsf{T}}\,\mathbf{B}_0 + \mathbf{v}_0^{\mathsf{T}} \tag{4}$$

From the accounting equations (3) and (4), it is usual to obtain the so-called *Leontief quantity model* and the *Ghosh price model*, respectively. However, we must include also two other types of models that are not so often treated in the input-output literature but that deserve to be mentioned for the sake of comprehensiveness.

*Quantity models*

Equation (3) rests on the assumption of fixed technical coefficients being the new industry (product) output vector ($\mathbf{x}_1$) required for an exogenously specified new final demand vector ($\mathbf{f}_1$) such that,

$$\mathbf{x}_1 = (\mathbf{I} - \mathbf{A}_0)^{-1}\,\mathbf{f}_1 \tag{5}$$

Given a shock in the physical amounts consumed by final users of a product (or of the bundle of products produced by a certain industry, both primarily and secondarily produced), then the effect on the total output value of the industry (product) output is given by $\mathbf{x}_1$. Notice that in this *Leontief quantity model* there is no change in prices.

Furthermore, equation (5) can also be expressed as a ratio per unit of output value of the period 0 as[2],

$$\hat{\mathbf{x}}_0^{-1}\mathbf{x}_1 = \hat{\mathbf{x}}_0^{-1}(\mathbf{I}-\mathbf{A}_0)^{-1}\hat{\mathbf{x}}_0\hat{\mathbf{x}}_0^{-1}\mathbf{f}_1 = (\mathbf{I}-\mathbf{B}_0)^{-1}\hat{\mathbf{x}}_0^{-1}\mathbf{f}_1 \tag{6}$$

which gives the variation rate of the quantities produced to meet the new final demand. That is, the new output total value ($\mathbf{x}_1$) results from the multiplication of old prices ($\mathbf{p}_0$) by the new quantities demanded ($\mathbf{q}_1$) such as,

$$\mathbf{x}_1 = \hat{\mathbf{p}}_0\mathbf{q}_1 \tag{7}$$

whilst the old output values result from the amounts consumed valued at prices of period 0, as

$$\hat{\mathbf{x}}_0^{-1} = (\hat{\mathbf{p}}_0\hat{\mathbf{q}}_0)^{-1} = \hat{\mathbf{q}}_0^{-1}\hat{\mathbf{p}}_0^{-1} \tag{8}$$

Then, by replacing the right-hand side (RHS) of equation (6) by equations (7) and (8), it is straightforward that,

$$\hat{\mathbf{q}}_0^{-1}\hat{\mathbf{p}}_0^{-1}\hat{\mathbf{p}}_0\mathbf{q}_1 = \hat{\mathbf{q}}_0^{-1}\mathbf{q}_1 = (\mathbf{I}-\mathbf{B}_0)^{-1}\hat{\mathbf{x}}_0^{-1}\mathbf{f}_1 \tag{9}$$

which is the so-called *Ghosh quantity model* (Dietzenbacher, 1997). A change in the final demand shares over the total output value of period 0 caused by variations in the quantities demanded will lead to changes in the quantities produced.

*Price models* Equation (4) is based on the assumption of fixed output coefficients. For a new value added vector ($\mathbf{v}_2^{\mathsf{T}}$), the new total output values are calculated by,

$$\mathbf{x}_2^{\mathsf{T}} = \mathbf{v}_2^{\mathsf{T}}(\mathbf{I}-\mathbf{B}_0)^{-1} \tag{10}$$

Given a price change in any of the primary factors used (generally speaking, capital and labour), then the effect on the output value of the industry (product) output is given by $\mathbf{x}_2$. Notice that in this *Ghosh price model* there is no change in quantities consumed of primary inputs and of goods and services.

Moreover, equation (10) can also be expressed as a ratio per unit of output value of the period 0 as,

$$\mathbf{x}_2^{\mathsf{T}}\hat{\mathbf{x}}_0^{-1} = \mathbf{v}_2^{\mathsf{T}}\hat{\mathbf{x}}_0^{-1}\hat{\mathbf{x}}_0(\mathbf{I}-\mathbf{B}_0)^{-1}\hat{\mathbf{x}}_0^{-1} = \mathbf{v}_2^{\mathsf{T}}\hat{\mathbf{x}}_0^{-1}(\mathbf{I}-\mathbf{A}_0)^{-1} \tag{11}$$

2. The relationship between the Leontief and the Ghosh inverses can be found in Miller and Blair (2009, p. 548).

which gives the price variation of products generated by the variation in the prices of primary factors. That is, the new output total value $(\mathbf{x}_2)$ results from the multiplication of old quantities produced $(\mathbf{q}_0)$ by the new prices $(\mathbf{p}_2)$ such as,

$$\mathbf{x}_2^{\top} = \mathbf{p}_2^{\top}\hat{\mathbf{q}}_0 \tag{12}$$

while the old output values result from the amounts consumed valued at prices of period 0, as

$$\hat{\mathbf{x}}_0^{-1} = (\hat{\mathbf{p}}_0\hat{\mathbf{q}}_0)^{-1} = \hat{\mathbf{q}}_0^{-1}\hat{\mathbf{p}}_0^{-1} \tag{13}$$

Therefore, by replacing the RHS of equation (11) by equations (12) and (13), it is easy to obtain that,

$$\mathbf{p}_2^{\top}\hat{\mathbf{q}}_0\hat{\mathbf{q}}_0^{-1}\hat{\mathbf{p}}_0^{-1} = \mathbf{p}_2^{\top}\hat{\mathbf{p}}_0^{-1} = \mathbf{v}_2^{\top}\hat{\mathbf{x}}_0^{-1}(\mathbf{I}-\mathbf{A}_0)^{-1} \tag{14}$$

which is the so-called *Leontief price model* or *supply-driven model* (Dietzenbacher, 1997). A change in value added shares over the total output value of period 0 caused by variations in the prices of primary inputs will lead to changes in product prices.

### 4.2. The relationship between the models and the choice of type of input-output table

*Quantity models*

The Ghosh and Leontief quantity models are demand driven models. They both measure the effects on the output (in physical and monetary values, respectively) of a change in final demand. To that purpose, the use of product-by-product tables would imply to assume a shock in the final demand of a specific product irrespectively of the industry that actually produced it. For instance, for an increase in the households' purchase of electric cars against fuel based vehicles one would need a product-by-product table in order to quantify the effects on the quantities of energy inputs supplied to meet such new demand. Furthermore, if greenhouse gas direct emissions are available on a product basis, the total effects on the environment can be easily calculated with a product-by-product table by multiplying the new output value $\mathbf{x}_1$ (from equation 5) by the emission levels per product output. Nevertheless, emission coefficients are mostly available on an industry basis, which then makes product-by-product tables unsuitable. Furthermore, if one eventually uses an industry-by-industry table the calculated effects would be caused instead by a change in the final demand of the bundle of goods and services produced by a specific industry, which is not necessarily that of a specific commodity. All in all, in the case of environmental analysis, the kind of data available and the objective of the analysis definitely play a major role in the choice of type of SIOT to be used.

Input-output analysis is also applied to labour market analyses through the calculation of employment multipliers under the Leontief quantity model. Due to the fact that employment data are usually recorded by firms and therefore grouped by industries, industry-by-industry tables may be more appropriate than product-by-product tables. It is not very likely to find employment data related to products. Moreover, one must bear in mind that the effects on employment thus calculated using industry-by-industry tables will be caused by a change in the final demand of a mixed bundle of goods and services produced by a certain sector, which does not necessarily be a single specific commodity.

The input-output quantity models are used to evaluate the effects of introducing a new product technology as well. Provided that the new technology refers to a single product and that it can be easily subtracted from its mother branch, the Leontief and Ghosh quantity models would allow for evaluating the effects on the output value (and physical amounts produced) of the other competing products. At this respect, product-by-product tables seems to be more suitable than industry-by-industry tables, where each industry produces more than one single product. Clearly, the new demand for a new product (e.g. electric cars) will drive a set of direct and indirect effects on the other product outputs.

The calculation of value added and income (wages and salaries) multipliers are also a matter of interest in the input-output literature. It is quite intuitive that the compensation of employees and the value added are clearly linked to industries rather than to products or homogenous branches. Industry-by-industry tables keep a direct link to the original statistical sources. Bearing this in mind, industry-by-industry tables are in this case also preferable to product-by-product tables although the IO literature admit several impact analyses on the basis of value added/income related to homogenous branches of activities.

As a summarizing remark, the IO quantity models are driven by changes in the amounts of goods and services consumed or demanded. The use of product-by-product tables is preferable since the shock can be easily assigned to a single product and the output effects can also be related to homogenous branches of activities. To the contrary, the use of industry-by-industry tables in this context would lead to measure the effects of a variation in the demanded quantity of a mixed bundle of goods and services produced by a certain industry on the industry output values and amounts of (mixed) goods and services produced. The choice favours clearly product-by-product tables almost in all cases. However, the Leontief quantity model is extensively used to account for many different kinds of multiplier effects, e.g. environmental, employment, income... that needs data that are almost solely available on an industry basis. To some extent, this justifies the use of industry-by-industry tables in some situations. Therefore, it seems to be a clear trade-off. Either one assumes that the additional data (environmental, employment, income...) is on a product basis and uses product-by-product tables to measure the effects on the output value (also in physical terms) of changes in final demand of single products, or one assumes that the additional data is on an industry basis and uses industry-by-industry tables, although being aware that the derived effects on total output values are referred not to single products but to a mixed bundle of goods and services produced by a certain industry.

*Price models*

The Ghosh and Leontief price models measure the effects of variations in the prices of primary inputs on the output value and on the prices of goods and services, respectively. The amount of factor inputs used remains unchanged and so the amounts of goods and services produced. These models are seen as supply-side driven models preferably to be used in cases of shortage of supply or excess of demand. Variations in salaries and wages per hour, in profit rates, in fixed capital use rates or in net tax rates[3] on production will generate changes in prices of goods and services and output value that could be quantified through the price IO models. As a result, industry-by-industry tables seems to be more suitable for these kind of analyses since initial changes are referred to the different components of the value added, which are directly linked to the surveyed firms data and/or groups of firms (industries) data. Indeed, statistical data on labour costs are referred to workers employed in industries and not in homogenous branches of activity. Environmentally oriented fiscal policies (excluding taxes on products) on taxes and subsidies on production (e.g. environmental tax) are commonly referred to the carbon emissions generated at the level of industries rather than to homogenous branches[4]. Moreover, profit rates are also related to firms and industries rather than to products.

Nevertheless, the price changes obtained through the IO price models using an industry-by-industry SIOT are not reflecting single product price variations but variations in the prices of a mixed bundle of goods and services produced by an industry. Hence, there is a clear trade-off again at this respect. Either one assumes that changes in primary inputs occur in homogenous branches and uses product-by-product tables to calculate single product price changes or one assumes that the price variations of primary factors occur in industries and uses industry-by-industry tables to obtain mixed product price changes. The choice is eventually up to the user.

*Supply-use tables*

Two major trade-offs have been identified concerning the choice of type of SIOT to be used in impact analysis. The main difficulty underlying the two trade-offs is referred to the symmetry of the SIOTs. They are defined as product-by-product or industry-by-industry type. Hence, if one is interested in estimating, for instance, the effects of an increase in the labour costs of the electricity sector (industry) on the prices of fuels

3. Generally speaking, the taxes less subsidies on production included in the value added at basic prices are those that are not payable per unit of some good or service produced or transacted (ESA95).

4. The ESA95 (4.22) includes taxes on pollution resulting from production activities as “other taxes on production” (D29); although, they may actually appear to be taxes on products (e.g. energy products). These pollution taxes consist of taxes levied on the emission or discharge into the environment of noxious gases, liquids or other harmful substances.

(product), then the choice of type of SIOT would lead to provide two different answers with neither of them being the correct one. On the one hand, if we use product-by-product tables we will be assigning the increase of labour costs to a homogenous branch of activity and not to the electricity sector and on the other hand, if we use industry-by-industry tables, the price effects will correspond to a mixed basket of goods and services of the fuel producing industry rather than to fuel.

To solve this issue, supply-use tables are clearly the best choice since they are defined on a product-by-industry basis rather than on a product or industry basis. However, there has been very little research on the application of supply and use tables to impact analysis. To the knowledge of the authors, the single contributions at this respect can be found in ten Raa and Rueda-Cantuche (2007) and in Rueda-Cantuche and Amores (2010). The former authors proved that employment and output multipliers (from the Leontief quantity model) can be derived from supply and use data by regressing employment (output) by industries on the net output[5] by products. Therefore, a change in the net output of products (implicitly a change in the final demand) will cause a variation in the employment (output) of industries. The interested reader may find more details in the cited paper. The latter contribution relates to environmental input-output impact analysis and applied the same concept to carbon dioxide emissions in Denmark. This line of research can be further extended methodologically to include time series of multiregional supply-use systems. So far it has been applied only to a single-country for one year only.

## 5. Conclusions and recommendations

This section summarizes the main conclusions and recommendations that can be drawn from the paper.

The construction of symmetric input-output tables (SIOTs) is a controversial issue in the input-output literature as regard the choice of model to construct both product-by-product and industry-by-industry SIOTs, especially the former ones. However, there has been so far little attention paid on the choice of type of SIOT to carry out impact analyses let alone other input-output applications. The UN and Eurostat systems of national accounts just simply refer to this issue vaguely and basically recommend nothing except that the purpose of the analysis will determine the choice of type to be used. Moreover, there are no explicit guidelines for the user to make the correct choice accordingly with its own purpose.

In empirical research, it depends on the objectives of the analysis which type of table is best suited for economic analysis. Particularly in impact analyses, questions

5. Ten Raa and Rueda-Cantuche (2007) defined net output as the difference between the intermediate parts of the supply and use matrices, which incidentally makes the final demand vector if one sums the elements of the net output matrix over columns.

like, for example, what fuel price effects would generate an increase in the labour costs of the electricity industry cannot really be answered by input-output price models as it is generally thought. Moreover, this is even independent of the type of SIOT used. Either one assumes that changes in primary costs (labour) occur in homogeneous branches rather than in industries and therefore uses product-by-product tables or one assumes that the price changes of primary factors effectively occur in industries and thus, uses industry-by-industry tables. Nonetheless, the corresponding reported price effects will be those of the fuel industry rather than those of the fuel product itself.

As regard input-output quantity models there is also a trade-off in the case of impact analyses related to environment, employment... or any economic dimension for which data is mainly available on an industry basis. Either one assumes that the additional data external to the input-output system (employment, emissions...) is on a product basis and uses a product-by-product table to evaluate the total effects of a change in the amount of the final demand consumed of a single product (like e.g. bio-fuels) or one assumes that the additional data is on an industry basis and uses industry-by-industry tables. Nevertheless, the derived total effects on employment, emissions... will correspond to a change in the output of a mixed bundle of goods and services produced by a certain industry rather than to changes in single product outputs.

Two major trade-offs have been identified concerning the choice of type of SIOT to be used in input-output impact analyses. The main shortcoming underlying this issue is related to the symmetry of SIOTs. They are defined as either product-by-product or industry-by-industry type. To solve this matter efficiently, supply and use tables are clearly the best choice since they are defined on a product-by-industry basis rather than solely on a product or industry basis. It is therefore advisable to follow the lines of the pioneering works of ten Raa and Rueda-Cantuche (2007) and Rueda-Cantuche and Amores (2010) and continue exploring the use of supply and use tables in the calculation of input-output impact multipliers of any kind. These authors currently propose to use econometric techniques to estimate unbiased and consistent input-output effects of any kind (emissions, employment, income...) from Model A and rectangular supply and use tables. This new approach opens up the door to further research with the other three models (B, C and D) and to provide possibly the first reliable inference based results in input-output analysis (including hypotheses tests, confidence intervals...). Of course, one can always come back to standard input-output analysis bearing in mind the methodological trade-offs addressed in this paper.

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