AN INDEX OF DIVERSITY IN STRATIFIED RANDOM SAMPLING BASED ON THE HYPOENTROPY MEASURE

L. PARDO. D. MORALES

Universidad Complutense de Madrid

We consider a measure of the diversity of a population based on the λ-measure of hypoentropy introduced by Ferreri (1980). Our purpose is to study its asymptotic distribution in a stratified sampling and its application to testing hypothesis. A numerical example based on real data is given.

Keywords: Asymptotic normality, Diversity, Hypoentropy measure, Estimation, Testing hypothesis, Stratified sampling.

A.M.S. subject classification: 62B10, 94A15.

1. INTRODUCTION

When the observations from a population are classified according to several categories, the uncertainty of the population may be quantified by means of several measures in Information Theory. The diversity of the population is intuitively intended as a measure of the average variability of classes in it, based on the number of classes and their relative frequencies.

Consider a finite population of N individuals which is classified according to a classification process of factor X into M classes or especies $x_1, \ldots, x_M$. We denote by $\mathfrak{X}$ the set of all categories or classes

$$\mathfrak{X} = \{x_1, \ldots, x_m\}$$

Let $\triangle_M = \left\{ P = (p_1, \ldots, p_M)/p_i \geq 0, \sum_{i=1}^{M} p_i = 1 \right\}$ be the set of all complete finite discrete probability distribution on the measurable space $(\mathfrak{X}, \beta_{\mathfrak{X}})$.

Rao (1982) established that a measure of diversity is a function

$$\Phi : \triangle_M \longrightarrow \mathbf{R},$$

satisfying the following conditions:

i) $\Phi(P) \geq 0 \; \forall P \in \triangle_M$ and $\Phi(P) = 0$ iff P is degenerate.

ii) Φ is a concave function of P in $\triangle_M$.

We shall refer to $\Phi(p)$ as the diversity measure within a probability space $(\mathfrak{X}, \beta_{\mathfrak{X}}, P)$. The condition i) is a natural one since a measure of diversity should be nonnegative and take the value zero when all individuals of a population are identical, i.e., when the associated probability measure is concentrated at a particular point of $\mathfrak{X}$. The condition (ii) is motivated by the consideration that the diversity in a mixture of populations should not be smaller than the average of the diversities within individual populations. Consequently, the entropy measures can be used as diversity indices.

Ferreri (1980) introduced a generalization of Shannon's entropy called hypoentropy; the expression of this measure of uncertainty for a discrete probability distribution $P \in \triangle_M$, is given by

$$\Phi_\lambda(P) = \left(1 + \frac{1}{\lambda}\right) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M} (1 + \lambda p_i) \log(1 + \lambda p_i), \; \lambda > 0$$

where

$$\lim_{\lambda \to \infty} \Phi_\lambda(P) = H(P) = - \sum_{i=1}^{M} p_i \log p_i$$

Assume that a sample of n members is drawn at random sampling. If we consider the estimate $\hat{\phi}_\lambda$ obtained by replacing p_i's by the observed proportions $\hat{p}_i$, $i = 1, \ldots, M$, then

$$n^{1/2} \left(\hat{\phi}_\lambda - \phi_\lambda(p_1, p_2, \ldots, p_M) \right) \xrightarrow[n \uparrow \infty]{L} \mathcal{N}(0, \sigma_\lambda^2)$$

where

$$\sigma_\lambda^2 = \sum_{i=1}^{M} p_i \log^2(1 + \lambda p_i) - \left(\sum_{i=1}^{M} p_i \log(1 + \lambda p_i) \right)^2$$

which has been used for testing several hypothesis (ref. Morales *et al.*, 1990).

Now we suppose that the population can be divided into r non–overlapping subpopulations, called strata, as homogeneous as possible with respect to the diversity associated with X. Let N_j be the number of individuals in the jth stratum (so that, $\sum_{j=1}^{r} N_j = N$), and let p_{ik} denote the probability that a randomly selected individual belongs to the kth stratum and to the class x_i ($i = 1, \ldots, M$, $k = 1, \ldots, r$). Thus, $\sum_{i=1}^{M} p_{ik} = N_k/N$, $\sum_{i=1}^{M} \sum_{j=1}^{r} p_{ij} = 1$. Let p_i be the probability that a randomly selected individual in the whole population will belong to the class x_i ($p_{i.} = \sum_{k=1}^{r} p_{ik}$, $i = 1, \ldots, M$). Then the hypoentropy population diversity associated with X is given by

$$
\begin{aligned}
\Phi_\lambda^s(X) &= (1 + \frac{1}{\lambda}) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M} (1 + \lambda p_{i.}) \log(1 + \lambda p_{i.}) \\
&= (1 + \frac{1}{\lambda}) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M} (1 + \lambda \sum_{k=1}^{r} p_{ik}) \log(1 + \lambda \sum_{k=1}^{r} p_{ik}), \ \lambda > 0
\end{aligned}
$$

Assume that a stratified sample of size n is drawn at random from the population independently in different strata. We hereafter suppose that the sample is chosen by proportional allocation in each stratum. Assume that a sample of size n_k is drawn independently at random with replacement from the kth stratum, where $n_k/n = N_K/N$. If f_{ik} denotes the relative frequency of individuals belonging to the class x_i an to kth stratum (and, hence $\sum_{i=1}^{M} f_{ik} = n_k/n$), and $f_i = \sum_{k=1}^{r} f_{ik}$, then the diversity sample with respect to the classification process or factor X could be quantified by means of the analogue estimates, the hypoentropy sample diversity, $\hat{\phi}_\lambda^s = \hat{\phi}_\lambda^s(X)$. Following the ideas in M.A.

13

Gil (1989), we will study in this paper asymptotic behavior of the hypoentropy sample diversity $\hat{\phi}_\lambda^s$.

2. ASYMPTOTIC DISTRIBUTION OF $\hat{\phi}_\lambda^s$

In this section, we state a general result concerning the asymptotic behavior of the hypoentropy sample diversity, $\hat{\phi}_\lambda^s$, in the stratified random sampling with proportional allocation, with replacement in each stratum and independence among different strata.

The following theorem establishes the asymptotic behavior of $\hat{\phi}_\lambda^s$.

Theorem 1

If we consider the estimate $\hat{\phi}_\lambda^s$, then

$$n^{1/2}\left(\hat{\phi}_\lambda^s - \phi_\lambda^s(X)\right) \longrightarrow \mathcal{N}(0, \sigma_{\lambda,s}^2)$$

where

$$\sigma_{\lambda,s}^2 = \frac{1}{N}\sum_{k=1}^r N_k\left(\sum_{i=1}^M t_{i.}^2 \frac{N}{N_k}p_{ik} - \left(\sum_{i=1}^M \frac{N}{N_k}p_{ik}t_{i.}\right)^2\right)$$

and $t_{i.} = -\log(1 + \lambda p_{i.}) - 1$.

Proof

Let us define

$$G_\lambda^s(f^*) = \hat{\phi}_\lambda^s(f)\left(\hat{\phi}_\lambda^s(f) \equiv \hat{\phi}_\lambda^s(X)\right)$$

where $f^* = (f_{11}, \ldots, f_{(M-1)1}, \ldots, f_{1r}, \ldots, f_{(M-1)r})$ and $f = (f_{11}, \ldots, f_{M1}, \ldots, f_{1r}, \ldots, f_{Mr})$. Now we consider the Taylor expansion for $G_\lambda^s(f^*)$

$$G_\lambda^s(f^*) = G_\lambda^s(p^*) + \sum_{i=1}^{M-1}\sum_{k=1}^r \frac{\partial G_\lambda^s(p^*)}{\partial p_{ik}}(f_{ik} - p_{ik}) + R_n$$

in a neighbourhood of

14

$$p^* = \left(p_{11}, \ldots, p_{(M-1)1}, \ldots, p_{1r}, \ldots, p_{(M-1)r}\right)$$

where

$$G_\lambda^s(p^*) = \phi_\lambda^s(p)(\phi_\lambda^s(p) \equiv \phi_\lambda^s(X)), \text{ and } p = (p_{11}, \ldots, p_{M1}, \ldots, p_{1r}, \ldots, p_{Mr})$$

and R_n is the Lagrange rest.

As,

$$
\begin{aligned}
G_\lambda^s(p^*) &= \left(1 + \frac{1}{\lambda}\right) \log(1 + \lambda) - \frac{1}{\lambda} \sum_{i=1}^{M-1} \left(1 + \lambda \sum_{k=1}^{r} p_{ik}\right) \log \left(1 + \lambda \sum_{k=1}^{r} p_{ik}\right) - \\
&\quad - \frac{1}{\lambda} \left(\left(1 + \lambda(1 - \sum_{k=1}^{r} \sum_{i=1}^{M-1} p_{ik})\right) \log \left(1 + \lambda(1 - \sum_{k=1}^{r} \sum_{i=1}^{M-1} p_{ik})\right)\right)
\end{aligned}
$$

we have

$$\frac{\partial G_\lambda^s(p^*)}{\partial p_{ik}} = -\left(\log(1 + \lambda p_{i.}) + 1\right) + \left(\log(1 + \lambda p_{M.}) + 1\right).$$

Then,

$$
\begin{aligned}
G_\lambda^s(f^*) &= G_\lambda^s(p^*) + \sum_{i=1}^{M-1} \left(-\log(1 + \lambda p_{i.}) - 1\right) (f_{i.} - p_{i.}) \\
&\quad + \sum_{i=1}^{M-1} \left(\log(1 + \lambda p_{M.}) + 1\right) (f_{i.} - p_{i.}) + R_n \\
&= G_\lambda^s(p^*) + \sum_{i=1}^{M-1} \left(-\log(1 + \lambda p_{i.}) - 1\right) (f_{i.} - p_{i.}) \\
&\quad + \left(\log(1 + \lambda p_{M.}) + 1\right) (1' - f_{M.} - (1 - p_{M.})) + R_n = \\
&= G_\lambda^s(p^*) + \sum_{i=1}^{M} \left(-\log(1 + \lambda p_{i.}) - 1\right) (f_{i.} - p_{i.}) + R_n
\end{aligned}
$$

15

Therefore, as

$$G_\lambda^s(p^*) = \phi_\lambda^s(p) \quad \text{and} \quad G_\lambda^s(f^*) = \hat{\phi}_\lambda^s(f)$$

it follows that, the random variables

$$\left(\hat{\phi}_\lambda^s - \phi_\lambda^s(X)\right) \quad \text{and} \quad \sum_{i=1}^M (-\log(1 + \lambda p_i) - 1)(f_{i.} - p_{i.})$$

converge in law to the same distribution because R_n converge in probability to zero. From now on, we denote $t_{i.} = (-\log(1 + \lambda p_{i.}) - 1)$.

The random vectors,

$$\left(\frac{n}{n_k} f_{1k}, \ldots, \frac{n}{n_k} f(M-1)k\right), \qquad k = 1, \ldots, r$$

are independent and distributed as a multinomial distribution of parameters $\left(n_k; \frac{N}{N_k} p_{1k}, \ldots, \frac{N}{N_k} p_{(M-1)k}\right)$, respectively.

Applying the $(M-1)$–dimensional Central Limit Theorem, we obtain

$$n_k^{1/2} \left(\left(\frac{n}{n_k} f_{1k} - \frac{N}{N_k} p_{1k}\right), \ldots, \left(\frac{n}{n_k} f_{(M-1)k} - \frac{N}{N_k} p_{(M-1)k}\right)\right) \underset{n_k \uparrow \infty}{\longrightarrow} \mathcal{N}(0, \Sigma(K))$$
$$k = 1, \ldots, r$$

where

$$\Sigma(K) = \begin{pmatrix} \frac{N}{N_k} p_{1k} & & 0 \\ & \ddots & \\ 0 & & \frac{N}{N_k} p_{(M-1)k} \end{pmatrix} - P(K)P(K)^t \qquad k = 1, \ldots, r,$$

and

$$P(K)^t = \left(\frac{N}{N_k} p_{1k}, \ldots, \frac{N}{N_k} p_{(M-1)k}\right),$$

i.e.,

16

$$n_k^{1/2} \frac{N}{N_k} \left((f_{1k} - p_{1k}), \ldots, (f_{(M-1)k} - p_{(M-1)k})\right) \xrightarrow[n_k \uparrow \infty]{L} \mathcal{N}(0, \Sigma(K))$$

$$k = 1, \ldots, r$$

as

$$n^{1/2} = n_k^{1/2} \left(\frac{N}{N_k}\right)^{1/2}$$

we have

$$n^{1/2} \left(\frac{N}{N_k}\right)^{1/2} \left((f_{1k} - p_{1k}), \ldots, (f_{(M-1)k} - p_{(M-1)k})\right) \xrightarrow[n_k \uparrow \infty]{} \mathcal{N}(0, \Sigma(K))$$

Therefore

$$n^{1/2} \left(\frac{N}{N_k}\right)^{1/2} \sum_{i=1}^{M-1} (t_{i.} - t_{M.})(f_{ik} - p_{ik}) \xrightarrow[n_k \uparrow \infty]{} \mathcal{N}(0, T^t \Sigma(K) T)$$

with

$$T = (t_{1.} - t_{M.}, \ldots, t_{M-1.} - t_{M.})^t$$

As,

$$\sum_{i=1}^{M-1} (t_{i.} - t_{M.})(f_{ik} - p_{ik}) = \sum_{i=1}^{M} t_{i.} (f_{ik} - p_{ik})$$

we have

$$n^{1/2} \sum_{i=1}^{M} \sum_{k=1}^{r} t_{i.}(f_{ik} - p_{ik}) =$$

$$= n^{1/2} \sum_{i=1}^{M} t_{i.}(f_{i.} - p_{i.}) \xrightarrow[n \uparrow \infty]{L} \mathcal{N}\left(0, \frac{1}{N} \sum_{k=1}^{r} N_k T^t \Sigma(K) T\right)$$

17

Therefore,

$$
\sigma_{\lambda,s}^2 = \frac{1}{N} \sum_{k=1}^{r} N_k T^t \Sigma(K) T =
$$

$$
= \frac{1}{N} \sum_{k=1}^{r} N_k \left(\sum_{i=1}^{M} t_i^2 \frac{N}{N_k} p_{ik} - \left(\sum_{i=1}^{M} \frac{N}{N_k} p_{ik} t_i \right)^2 \right)
$$

3. APPLICATIONS ON TESTING HYPOTHESIS

In the stratified radom sampling with proportional allocation with replacement in each stratum and independence among different strata, the hypoentropy can be used for testing the following hypothesis:

i) $H_o : \phi_\lambda^s = D_O$ against $\phi_\lambda^s > D_O$. Under H_o, the statistic

$$
Z = \frac{n^{1/2} \left(\hat{\phi}_\lambda^s - D_O \right)}{\sigma_{\lambda,s}}
$$

has aproximately a standard normal distribution for sufficiently large n. Clearly, we reject H_o at level α if $z > z_\alpha$, where z_α is such that $P(Z \geq z_\alpha) = \alpha$. Similar arguments may be applied in the remaining cases, i.e., $H_1 : \phi_\lambda^s < D_O$ and $H_1 : \phi_\lambda^s \neq D_O$.

On the basis of this result we can obtain a method to test the null hypothesis:

$$
H_o : p_{1.} = p_{2.} = \cdots = p_{M.} = 1/M
$$

This hypothesis is equivalent to test

$$
H_o : \phi_\lambda^s = \phi_{\lambda,MAX}^s
$$

where

$$\phi^s_{\lambda,\,\text{MAX}} = (1 + 1/\lambda)\log(1 + \lambda) - \frac{M + \lambda}{\lambda}\log(1 + \lambda/M).$$

In this case we reject H_o at the level α if

$$\hat{\phi}^s_\lambda < \phi^s_{\lambda,\,\text{MAX}} - \frac{z_{\alpha/2}\hat{\sigma}_{\lambda,s}}{n^{1/2}} \quad \text{or} \quad \hat{\phi}^s_\lambda > \phi^s_{\lambda,\,\text{MAX}} + \frac{z_{\alpha/2}\hat{\sigma}_{\lambda,s}}{n^{1/2}}$$

Since in the non–null case

$$\frac{n^{1/2}[\hat{\phi}^s_\lambda - \phi^s_\lambda(P)]}{\sigma_{\lambda,s}(P)} \underset{n \uparrow \infty}{\longrightarrow} \mathcal{N}(0,1),$$

the asymptotic power function in $P = (p_1, p_2, \ldots, p_M)$ is given by

$$
\begin{aligned}
\beta_\lambda(P) \;=\;& \Psi\left(\frac{n^{1/2}(\phi^s_{\lambda,\,\text{MAX}} - \phi^s_\lambda(P)) - \sigma_{\lambda,s}z_{\alpha/2}}{\sigma_{\lambda,s}(P)}\right) + 1 \\
&- \Psi\left(\frac{n^{1/2}(\phi^s_{\lambda,\,\text{MAX}} - \phi^s_\lambda(P)) + \sigma_{\lambda,s}z_{\alpha/2}}{\sigma_{\lambda,s}(P)}\right)
\end{aligned}
$$

where Ψ denotes $P(X \leq x)$ when X is normally distributed with mean 0 and variance 1.

ii) $H_o : D_1 = D_2$ (diversities of two independent populations are equal) against one–sided or two sided alternatives. Now under H_o, the statistic

$$Z = \frac{(n_1 n_2)^{1/2}\,(\hat{\phi}^s_{\lambda,1} - \hat{\phi}^s_{\lambda,2})}{\left(n_2\hat{\sigma}^2_{\lambda,s,1} + n_1\hat{\sigma}^2_{\lambda,s,2}\right)^{1/2}}$$

has aproximately a standard normal distribution, where subscript i has been used to denote population i and n_i denote the sample size in population i, $(i = 1, 2)$.

Remark

From theorem 1, an approximate $1 - \alpha$ level confidence interval for ϕ_λ^s is given by

$$\left(\hat{\phi}_\lambda^s - \frac{\hat{\sigma}_{\lambda,s} z_{\alpha/2}}{n^{1/2}}, \ \hat{\phi}_\lambda^s + \frac{\hat{\sigma}_{\lambda,s} z_{\alpha/2}}{n^{1/2}} \right) ;$$

furthermore, the minimum sample size guaranteeing a specified limit of error ϵ with a small risk is

$$n = \left[\frac{\hat{\sigma}_{\lambda,s}^2 z_{\alpha/2}^2}{\epsilon^2} \right] + 1$$

4. CONNECTIONS WITH THE RANDOM SAMPLING

Stratification provides a method of utilizing supplemental information to get greater precision in our sample estimates. Auxiliary information may be used to divide the population into groups, called strata, such that the elements within each group are more alike than are the elements in the population as a whole. Then a sample is selected from each stratum and the sample results from the different strata are pooled in order to arrive at an estimate for the whole. If there are large differences between the units in the differents strata the accuracy of the averall estimates will be substantially increased as strata will be represented in their correct proportions, whereas in a random sample these proportions will be subject to sampling errors. We are now going to formulate the comments above for the asymptotic variances as well as for their analogue estimates.

The following theorem establish that the asymptotic variance of the statistic $n^{1/2}(\hat{\phi}_\lambda^s - \phi_\lambda^s(X))$ is smaller than the asymptotic variance of $n^{1/2}(\hat{\phi}_\lambda - \phi_\lambda(P))$.

Theorem

It is verified that

$$\sigma_{s,\lambda}^2 \leq \sigma_\lambda^2$$

with equality if and only if $r = 1$ or

$$\sum_{i=1}^{M} \log\left(1 + \lambda p_{i.}\right) \frac{N}{N_k} p_{ik}$$

does not depend on k $(k = 1, \ldots, r)$.

Proof

Since

$$\sigma_{\lambda,s}^2 = \frac{1}{N} \sum_{k=1}^{r} N_k \left(\sum_{i=1}^{M} t_i^2 \frac{N}{N_k} p_{ik} - \left(\sum_{i=1}^{M} \frac{N}{N_k} p_{ik} t_{i.} \right)^2 \right)$$

with $t_{i.} = \log(1 + \lambda p_{i.}) - 1$, we have

$$
\begin{aligned}
\sigma_{\lambda,s}^2 &= \frac{1}{N} \sum_{k=1}^{r} N_k \left(\sum_{i=1}^{M} (\log(1 + \lambda p_i) + 1)^2 \frac{N}{N_k} p_{ik} - \right. \\
&\quad - \left. \left(\sum_{i=1}^{M} \frac{N}{N_k} p_{ik}(\log(1 + \lambda p_i) + 1) \right)^2 \right) = \\
&= \sum_{k=1}^{r} \sum_{i=1}^{M} \log^2(1 + \lambda p_i) p_{ik} - \frac{1}{N} \sum_{k=1}^{r} N_k \left(\sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_{i.}) \right)^2 = \\
&= \sum_{i=1}^{M} \log^2(1 + \lambda p_i) p_i - \frac{1}{N} \sum_{k=1}^{r} N_k \left(\sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_{i.}) \right)^2
\end{aligned}
$$

Now we consider the random variable Z, taking the values

$$\sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_i), \quad k = 1, \ldots, r.$$

with probabilities $\frac{N_k}{N}$, respectively. Let us consider the convex function,

$$\phi(x) = x^2$$

21

then by using Jensen's inequality, we have

$$\left(\sum_{i=1}^{M} \log(1 + \lambda p_{i.}) p_{i.} \right)^2 \leq \frac{1}{N} \sum_{k=1}^{r} N_k \left(\sum_{i=1}^{M} \frac{N}{N_k} p_{ik} \log(1 + \lambda p_{i.}) \right)^2$$

therefore,

$$\sigma_{s,\lambda}^2 \leq \sigma_{\lambda}^2$$

5. A NUMERICAL EXAMPLE

In the Natural Farm Survey of England and Wales (Ministry of Agriculture, 1944, G) for the country of Hereford the farms sample were classified into the following domains:

	Percentage of arable land
A Mainly grass	0 – 29.9
B Intermediate	30 – 49.9
C Mainly arable	50 – 100

The population of farms over 25 acres was divided into three size–groups (25–100, 100–300, over 300). The basic data is given in Table I.

TABLE I

Size group (acres)	A	B	C	TOTAL
25 – 100	72	62	39	173
100 – 300	79	55	108	242
over 300	13	61	25	99

We use data from Table I to test for deviation from uniformity for $\alpha = 0.05$ and $\lambda = 10$. As

22

$$\phi_{\lambda}, \text{MAX} = 0.73145, \ \hat{\phi}_{\lambda} - \frac{\hat{\sigma}_{\lambda,s}1.96}{n^{1/2}} = 0.51660, \ \hat{\phi} + \frac{\hat{\sigma}_{\lambda,s}1.96}{n^{1/2}} = 0.89854$$

we have,

$$0.51660 < 0.73145 < 0.89850.$$

Therefore, we do not reject $H_o : p_A = p_B = p_C$.

In the following tables we have computed powers of test deviation from uniformity for $M = 3, r = 3$ and $\alpha = 0.05$, when there are 130 elements in the first stratum, 110 elements in the second stratum and 140 elements in the third stratum. We calculate powers of test $H_o : p_1 = p_2 = p_3 = 1/3$, i.e., the population is homogeneous, for different sets of probabilities (p_1, p_2, p_3), where p_i is the probability of class $x_i (i = 1, 2, \ldots, M)$. We consider the following values of $\lambda : .01, .1, 1, 10, 100, 1000$ and 10000. Observe that results for ϕ_{1000} can be interpreted as results for Shannon entropy.

λ	p_i.		
	0.29474	0.36842	0.33684
0.010		0.15971	
0.100		0.15085	
1.000		0.15003	
10.000		0.14941	
100.000		0.14913	
1000.000		0.14909	
10000.000		0.14908	

λ	p_i.		
	0.77368	0.16579	0.06053
0.010		1.00000	
0.100		1.00000	
1.000		1.00000	
10.000		1.00000	
100.000		1.00000	
1000.000		1.00000	
10000.000		1.00000	

λ	p_i. 0.46053	0.42105	0.11842
0.010		0.99999	
0.100		1.00000	
1.000		0.99999	
10.000		0.99997	
100.000		0.99993	
1000.000		0.99991	
10000.000		0.99991	

λ	p_i. 0.06579	0.07368	0.86053
0.010		1.00000	
0.100		1.00000	
1.000		1.00000	
10.000		1.00000	
100.000		1.00000	
1000.000		1.00000	
10000.000		1.00000	

λ	p_i. 0.32368	0.32895	0.34737
0.010		0.43509	
0.100		0.06137	
1.000		0.06009	
10.000		0.06007	
100.000		0.06008	
1000.000		0.06009	
10000.000		0.06010	

From these tables we see that the asymptotic powers of these seven tests are not very different. Also we observe that no test is uniformly better than the others.

6. ACKNOWLEDGMENT

7. REFERENCES

[1] **Ferreri, C.** (1980). "Hypoentropy and related heterogeneity, divergence and information measures". *Statistica, anno XL*, 2, 155-167.

[2] **Gil, M.A.** (1989). "A note on stratification and gain in precision in estimating diversity from large samples". *Commun. Statist. (Theory and Methods)* 18(4), 1521-1526.

[3] **Morales, D., Pardo, L, & Taneja, I.J.** (1990). "Hypoentropy as an index of diversity". Proceeding of the 2nd World Congress of the Bernouilli Society.

[4] **Nayak, T.P.** (1983). "Applications of entropy functions in measurement and analysis of diversity". Ph. D. thesis. Un. Pittsburgh. Ed. Un. Microfilm International.

[5] **Pardo, L., Morales, D. & Taneja, I.J.** "λ–measures of hypoentropy and comparison of experiments: Bayesian approach". Communicated.

[6] **Rao, C.R.** (1982). "Diversity and dissimilarity coefficients: a unified approach". *Theo. Popln. Bio.*, 21, 24-43.

[7] **Taneja, I.J.** (1986). "λ-Measures of hypoentropy and their applications". *Statistica, anno XLVI*, 4, 465-478.

[8] **Taneja, I.J., Pardo, L. & Morales, D.** "λ–Measures of hypoentropy and comparison of experiments: Blackwell and Lhemann approach". Communicated.